TUNING SYSTEMS — STRUCTURE

AND COMPARATIVE ANALYSIS

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TUNING SYSTEMS – STRUCTURE AND COMPARATIVE ANALYSIS

CHAPTER ONE

Introduction

For the purposes of this essay I define Music as "intentionally organized sound patterns". The difference between music and noise is therefore that noise is random. (Although some twentieth century composers incorporate randomness in their music, the very intention to do this makes it organized.)

Every sound, even noise, can be resolved, (split up) into simple tones, each being characterized by a certain pitch. Therefore even music composed of non-pitched percussion incorporates tones with particular pitches. In all music known to us the pitches used are somehow related. This very relation I define as a tuning system.

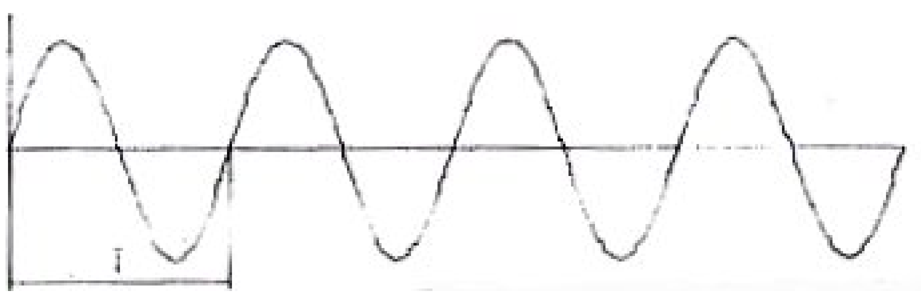
There is a large variety of tuning systems in world music. Some of them, are of extreme importance and have played a major role in the history of music. In this essay I will discuss how they are built,, how they compare to each other and what their importance in the history of music is.

The differences in the tuning systems used are more evident in keyboard and fretted instruments, because they do not allow the performer to deviate the pitch. This is the reason why in this essay the emphasis is placed on the piano, the organ, and the harpsichord.

The discussion of tuning systems is important, because they are a major factor to be considered in the characterization of a certain music epoch, for they are what determines the way music is perceived. Very few people have actually asked themselves why there are seven white and five black keys in an octave of a piano keyboard, or why some intervals sound consonant while others do not. I will try to answer these questions and I will emphasize the primary tuning systems of the past, the one used today and a new one, designed by myself. In this chapter I will give a brief introduction to the physical and mathematical principles used in the essay.

Sound (signal) is the periodic change of pressure of air, which man's sound analyzer, the ear, detects and interprets as sound. It is a mechanical wave, whose main characteristic is its frequency. The frequency of a sound is the number of times one period occurs in a second, and its unit is the Hertz (Hz). (The period of a wave is the time after which it starts repeating itself.) The graph of air pressure vs. time for a sound looks like this:

pressure



time

Fig. 1.1

Frequency is in fact what determines the pitch: the higher the frequency, the higher the pitch. For example, "A" below middle "C" has a frequency of 220 Hz, while "A" above middle "C" has a frequency of 440 Hz.

Let us consider a combination of two tones. The resultant signal is the sum of the two initial tones, and this is what the ear perceives when listening to them simultaneously. The fact that one hears them as separate ones is explained by a property of the ear to separate complex sound into its basic ingredients and to send the information to the brain in this form. The actual summation of the two tones is done by adding their pressure values for each instant of time (see Fig.1.2).

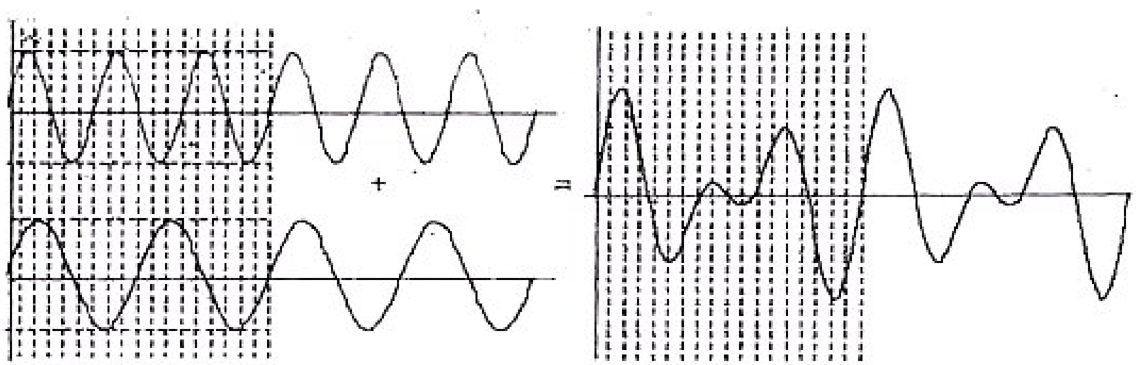


Fig 1.2

The intervallic relationship (interval width) between two tones can be mathematically expressed as the ratio of their frequencies. I will use the letter "?" for frequency. Let us now consider three different combinations of two tones with frequencies Fl and F2:

1) F2 = 2 F1

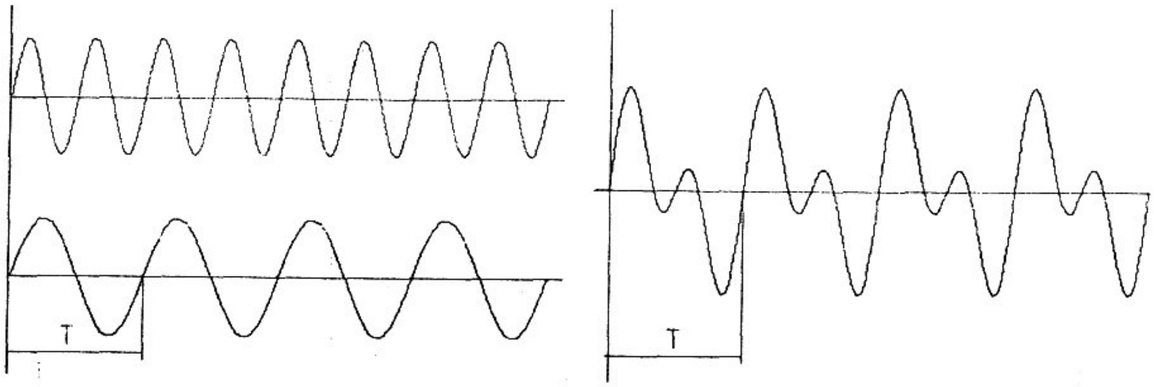


Fig. 1.3

2) F2 = 17/18 F1

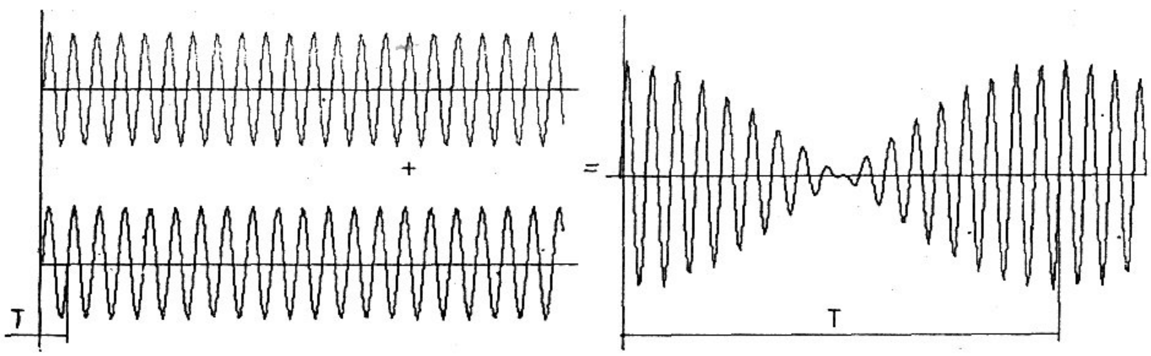


Fig. 1.4

3) F2 = 3/2 F1

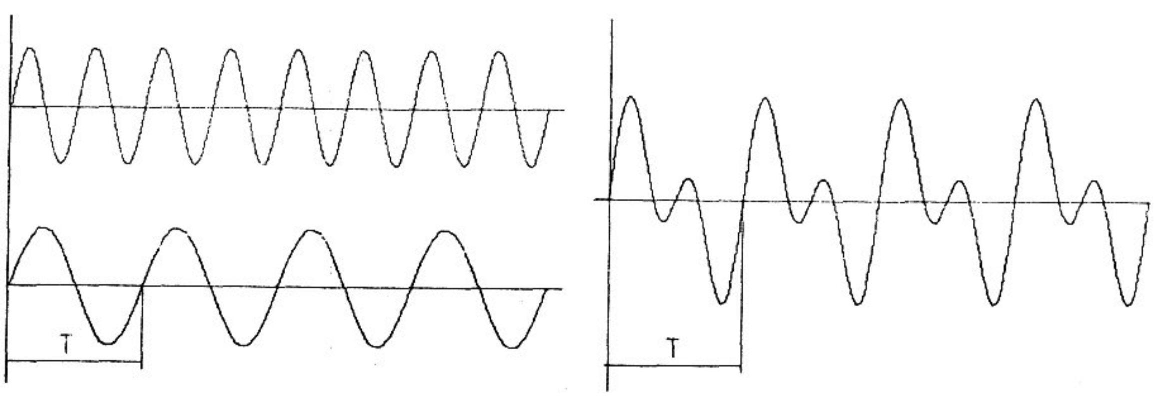


Fig. 1.5

In music these ratios correspond to the following intervals: 1) perfect octave; 2) minor second; 3) perfect fifth. A closer examination of the three resultant signals shows that the first one forms a smooth pattern with a period. equal to that of the first tone. One would, therefore, expect it to sound pleasant to the ear. In fact, that is the case: the perfect octave sounds very consonant, i.e. "agreeable".

The second combination forms a rather irregular pattern, which "beats". This beating sound is due to its period being eighteen times larger than the one of the first tone(1). The resultant signal is not pleasant to the ear; it is dissonant.

The third combination forms a quite regular .pattern, with a period twice, that of the first tone. It sounds consonant, although not as much as the first one.

After considering these examples one can conclude, that the closer the period of the sum of the two tones to that of the first one, the more consonant the combination. Mathematically speaking, only these combinations of two tones whose frequencies form ratios expressible as ratios of small integers (simple ratios)(2), sound consonant. Moreover, the smaller the integers, the more consonant the sound. For example, the perfect octave corresponds to a ratio of 2:1; the perfect fifth — to 3:2 (e.g. "c" to "g" = 392 Hz:262 Hz = 1.5 = 3:2); the perfect fourth — to 4:3; the

(1) In fact F beating = F2 – F1
(2) This comes from the fact, that the period of the resultant signal is equal to the least common multiple of the periods of the two tones, which is closer in value to them when they are in simple ratios

major third — to 5:4, etc. It *is* important to note that when two intervals are added to make a wider one, its ratio is the product of the ratios of the initial two, not their sum.

This is the underlying principle for the creation of any tuning system. However, some problems arise if one tries to build a tuning system wholly on this principle. That is why there is such a large variety of. tuning systems: each system considers different factors as most important.

In the following chapters I will use the letter "c" for cent, and the letters "M", "m" , "P" , "d", and "A" for "major", "minor", "perfect", "diminished", and "augmented" intervals respectively, followed by a number indicating the interval's width. (For example, d6 = diminished sixth.) Also "#" stands for sharp and "b" stands for flat.

CHAPTER TWO

Classic[1] Tuning Systems

All classic tuning systems have one thing in common; none are based on equal temperament, i.e. not all of the smallest intervals of the scale are of equal width. As a result only the tonic key is in tune, and the further away one modulates from it, the more out of tune the keys become. Why this is true will be discussed later. The most consonant interval, P8 , is kept perfectly in tune in ail tuning systems, because it *is* the basis for the building of the scales. In fact, P8 is so consonant, that two tones a PS apart are given the same letter name (e.g. A, a, a', etc.). Therefore, since P8 is present in all tuning systems, they differ from each other in the way an octave is subdivided. The tones of the other octaves above and below are formed by doubling or halving the frequencies of the tones within the basic octave, that correspond to them.

2.1 The Pythagorean Tuning System

This is one of the two ancient Greek tuning systems, chiefly used in the Middle Ages (tenth to thirteenth centuries). It was created by a group of philosophers, referred to as "the Pythagoreans". I will try to follow their thoughts.

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(1) The word "classic" when used for tuning systems stands for "ancient", i.e. worked out by the Ancient Greeks.

To create a tuning system one has to divide the octave into smaller intervals, with the frequencies of the so formed tones expressed with respect to a basic one with frequency Fl. After P8 (2:1), the most consonant interval has a ratio of 3:2. That is how a tone with frequency F5 = 3/2 Fl, called "the fifth", appeared in the Pythagorean tuning system. After this it is natural to look for a tone, with respect to which the basic tone is the fifth. It has a frequency of 2/3 Fl. However, since tones only within one octave are considered, their frequencies should be within the range [Fl, 2 Fl]. Therefore, to shift this tone to the necessary octave, its frequency is doubled, to give the result F4 = 4/3 Fl. This tone is called "the fourth". The ratio between the so formed fifth and fourth is 3/2 : 4/3 = 9/8. This very ratio was chosen by the Pythagoreans as the basic step of their scale.

Now the whole Pythagorean scale can be built: F2 = 9/8 F1, (the second); F3 = 9/8 F2 = 81/64 F1, (the third). If one continued in a similar way, one would obtain a tone higher than the fourth. Therefore, the result F4 = 4/3 F1 is kept, as well as F5 = 3/2 F1 , (the fifth). To continue further, F6 = 9/8 F5 = 27/16 F1, (the sixth); and F7 = 9/8 F6 = 243/127 F1, (the seventh); after which the octave (F8 = 2 F1) comes. Here is the list of ratios for the Pythagorean scale:

$$1\ ;\ \frac{9}{8}\ ;\ \frac{81}{64}\ ;\ \frac{4}{3}\ ;\ \frac{3}{2}\ ;\ \frac{27}{16}\ ;\ \frac{243}{128}\ ;\ 2$$

Let us now consider the so obtained tuning system, in detail. All the fifths are perfectly in tune, with a ratio of 3/2. Between the consecutive degrees of this scale there are two different intervals: M2, with ratio 9/8, and m2, with ratio a 256/243. m2 occurs between the third and the fourth, and between the seventh and the octave; while the rest are M2. M2 is not exactly twice as large as m2, but it is 2.26 times larger. The M2's in the Pythagorean system are the largest among all tuning systems, while the m2's are the smallest. Melodically, this fact is of great potential expressive value, because of the "handsomeness" of the big M2's and the "incisiveness" of the small m2's. The third is expressed in quite large integers (81/64), and therefore one would expect it to sound a bit out of tune. Although for a melody these slight "faults" do not make a big difference, the "bad" third becomes the reason for a rather unsatisfactory harmony. An attempt to "correct" this third lead to the appearance of

2.2 Just Intonation

Just Intonation, also called the "Ptolemaic system", appeared in the fourteenth century. In it the 81/64 third was replaced by the 5/4 third, which had a more consonant sound; (the ratio is much "simpler"). It is lower than the 81/64 one by the ratio of 5/4 : 81/64 = 80/81. This very amount is called "the comma of Didymus". The 27/16 sixth was lowered by a comma as well, to become 5/3, also much more consonant. The seventh changed a bit too, following the change of the sixth. It changed from 243/128 to 15/8.

Now the ratios look simpler:

$$1 \; ; \; \frac{9}{8} \; ; \; \frac{5}{4} \; ; \; \frac{4}{3} \; ; \; \frac{3}{2} \; ; \; \frac{5}{3} \; ; \; \frac{15}{8} \; ; \; 2$$

Since both the third and the sixth were lowered, the A-E fifth remained perfectly consonant, but the D-A fifth became a comma too flat. As I pointed out before, the P5 is the most important interval and it has to be in tune. This is why the D-A fifth was a serious flaw of Just Intonation. In addition, modulations in this system were difficult to make, because the quintal circularity was ruined by the dissonant D-A fifth. These major problems were probably the reason why this system was never quite accepted in practice.

Both of the classic tuning systems have one major disadvantage: not all semitones of the scale are of equal width. Therefore there are some enharmonic problems: in the Pythagorean system, for example, F# differs from Gb by 23 cents.(1 cent (c) is one hundredth of the contemporary semitone, or 1/1200 of the octave.) For the stringed instruments, (and the trombone), it is easy to play that difference, but for a keyboard instrument it is impossible. As a result theorists began looking for a tuning system, which would compromise between the different needs, so that no fault in it would be too large. Enharmonism, (i.e. when C# is the same as Db, D# = Eb, etc.), should also be present in it, because it is of primary importance for the developing harmony.

CHAPTER THREE

Temperament

3.1 Mean - Tone Temperament

Temperament is first mentioned in 1496, by the Italian theorist Franchino Gafori in the treatise Practica musica. In this system, the sharp dissonance of the D-A fifth of Just Intonation is spread out among the other fifths so that all of them are a bit out of tune, but overall satisfactory. All the major thirds in this system are perfectly in tune, with a ratio of 5/4. This is achieved by dividing the interval of two octaves and a M3, corresponding to a ratio of 5:1, into four P5's, each of them with a ratio of $\sqrt[4]{5}$:1. This ratio is smaller than the perfectly consonant 3:2 one by 5 c. (one quarter of a comma). Here is the list of ratios for this system:

$$1 \; ; \; \frac{\left(\sqrt[4]{5}\right)^2}{2} \; ; \; \frac{5}{4} \; ; \; \frac{2}{\sqrt[4]{5}} \; ; \; \sqrt[4]{5} \; ; \; \frac{\left(\sqrt[4]{5}\right)^3}{2} \; ; \; \frac{5\sqrt[4]{5}}{4} \; ; \; 2$$

Although in Mean - Tone Temperament only the octaves and the M3's are in tune, it sounds better melodically than Just Intonation. This is because all M2's In the scale are the same, and exactly equal to one half of the perfectly consonant M3. The name "Mean - Tone Temperament" comes from the averaging of all the M2's and M5's.

This tuning system may seem quite satisfactory at first, but the lack of enharmonism still remains a serious problem. Since there is only one black key between two white keys, on a keyboard instrument it is impossible to make a difference between two enharmonic notes, e.g. D# and Eb. As a result the keyboard instruments using Mean – Tone Temperament were tuned in the following way: C, C#, D, Eb, E, F, F#, G, G#, A, Bb, B, C. Therefore composers could not use other chromatically altered notes except these listed above. Any attempt to use Ab, for example, would lead to a severe dissonance. The fifth Ab-Eb, (when the key is tuned as G#, not as Ab), is nearly two commas off the perfectly consonant 3/2 one. This fifth was much worse than D-A of Just Intonation, and it was given a special name: "the wolf". This major disadvantage bothered the musicians and many different ways to avoid it were tried. For example, some organs were built with split keys for D# and Eb, and for G# and Ab. This certainly improved the sound of the instrument, but created difficulties for the performer. Another way to deal with "the wolf" was to distribute it among the other fifths, which finally lead to the creation of Equal Temperament in the seventeenth century.

3.2 Equal Temperament

In Equal Temperament all the m2's are exactly equal to one another and to one half of M2. In other words the octave was divided into 12 equal semitones, each corresponding to a ratio of $\sqrt[12]{2}$, or 100 c. wide. Since $\sqrt[12]{2}$ is an irrational number, the only interval that is perfectly in tune is the octave. All of the rest are slightly out of tune. The ratios of the intervals of the scale and the number of cents they are off the perfectly consonant ones are listed below.

Chart 3.1

Interval:	P1	M2	M3	P4	P5	M6	M7	P8
Ratio:	$2^{\frac{0}{12}}=1$	$2^{\frac{2}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{7}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{11}{12}}$	$2^{\frac{12}{12}}=2$
# c. off:	0	4	14	2	2	16	12	0

As one can see, the interval most out of tune, the. M6, is only 16 c. off. For some keyboard instruments, such as piano and harpsichord, this is of minor significance, because the nature of their timbre is one including some dissonance. In fact, a lot of professional piano tuners have been found to tune the one or two strings corresponding to the same key a bit out of tune. For the organ, however, this imperfection of Equal Temperament has some importance, because the organ is one of the instruments with "purest" sound.

The great advantages of the equally tempered system are unquestionable. First, it is the only tuning system, in which all the intervals with the same name have exactly the same width (e.g. P5, M3, etc.). This allows modulation to any key, with the new key just as much in tune as the previous .one, which is of extreme importance for the expanded, harmonies of the nineteenth and the twentieth centuries. Melodically, it has the advantage of equal steps in stepwise motion.

There are many musicians with negative opinions towards Equal Temperament. They say that if all twelve keys of the scale are identical the diversity of expression is lost. In the other tuning systems each tonality sounds different, because the tuning of the thirds and the fifths is not the same for all keys. The most commonly spread opinion was that "people would take little pleasure from transposition if all keys were the same". In 1726 Rameau made the following remarks:

> "...It is good to note that we receive different impressions from intervals in keeping with their different [degree of] alteration. For example, the major third, which [in its] natural [state] excites us to joy, as we know from experience, impresses upon us ideas even of fury when it is too large; and the minor third, which [in its] natural [state] transports us to sweetness and tenderness, saddens us when it is too small. Knowledgeable musicians know how to exploit these different effects of the intervals, and give value, by the expression they draw therefrom, to the alteration which one might [otherwise] condemn."

Later, in 1737, however, Rameau himself expresses the following idea:

> "He who believes that the different impressions which he receives from the differences caused in each transposed mode by the temperament [now] in use heighten its character and draw greater variety from it, will permit me to tell him that he is mistaken. The sense of variety arises from the intertwining of the keys and not at all from the alteration of the intervals, which can only displease the ear and consequently distract it from its functions.”

In the course of time this tuning system has been most widely recognized by musicians, and it is the one used today.

CHAPTER FOUR

Tuning Systems in the History of Music: Role And Comparison

Tuning systems have been more important in the development of music than one usually tends to think they have. For example, in medieval music of the tenth to the thirteenth centuries, the only intervals used were P1, P8, P4 and P5; thirds and sixths were considered dissonances. At that time, however, they were dissonances, because the Pythagorean system was used, in which M3 and M6 corresponded to the clumsy ratios of 81/64 and 27/16, which are far from being "consonant small-integer ratios". In that system the only tolerable intervals are P4, P5 and P8, and that is why organum was the only "harmony" used.

The first use of thirds and sixths chronologically coincided with the appearance of Just Intonation, because in this system they are perfectly in tune. In fact, some harmony based on triads was possible, but the sharply dissonant D-A fifth never allowed it to develop very much. Not all instruments have passed through all of these tuning systems. When Just intonation had become intolerable, the instruments diverged into two different directions: the keyboard instruments, such as the organ and the harpsichord, started using Mean - Tone Temperament; while the fretted instruments, such as the lute, viol and guitar, jumped straight into Equal Temperament. This is due to the fact, that if the frets were not based on this system, their positioning would result in mistuned unisons and octaves between the different strings of the instrument. (A very complicated system of frets was designed for fretted instruments using just intonation, see Fig. 4.1. This very existence of Equal Temperament in lutes as early as the sixteenth century made it possible for some composers, like Carlo Gesualdo, to use plenty of chromaticisms in their music.

Since at that time keyboard instruments were part of the orchestra, its unification was somewhat delayed by the difference *of* tuning systems used by the fretted and the keyboard instruments. Severe dissonances resulted from simultaneous playing of these two kinds of instruments. However, the later displacement of viols and lutes for the violin family eliminated this problem, since the lack of frets made the use of Equal Temperament unnecessary. This was, in a sense, a step backwards, because it lead to the universal use of Mean - Tone Temperament, which is considered "worse" than Equal Temperament today.

As mentioned in Chapter Three, Mean - Tone Temperament's major problem was the lack of enharmonism, due to which the chromatic degrees of the scale, (the black keys), were tuned to that only one of the two possibilities, which was more likely to be used in the given key. For example, in the key of C major, the black key between C and D was tuned to C#, rather than Db. This problem made it necessary to retune the harpsichord every time the basic key of the piece was changed. The harpsichord is easy to retune, but for the organ this is impossible.

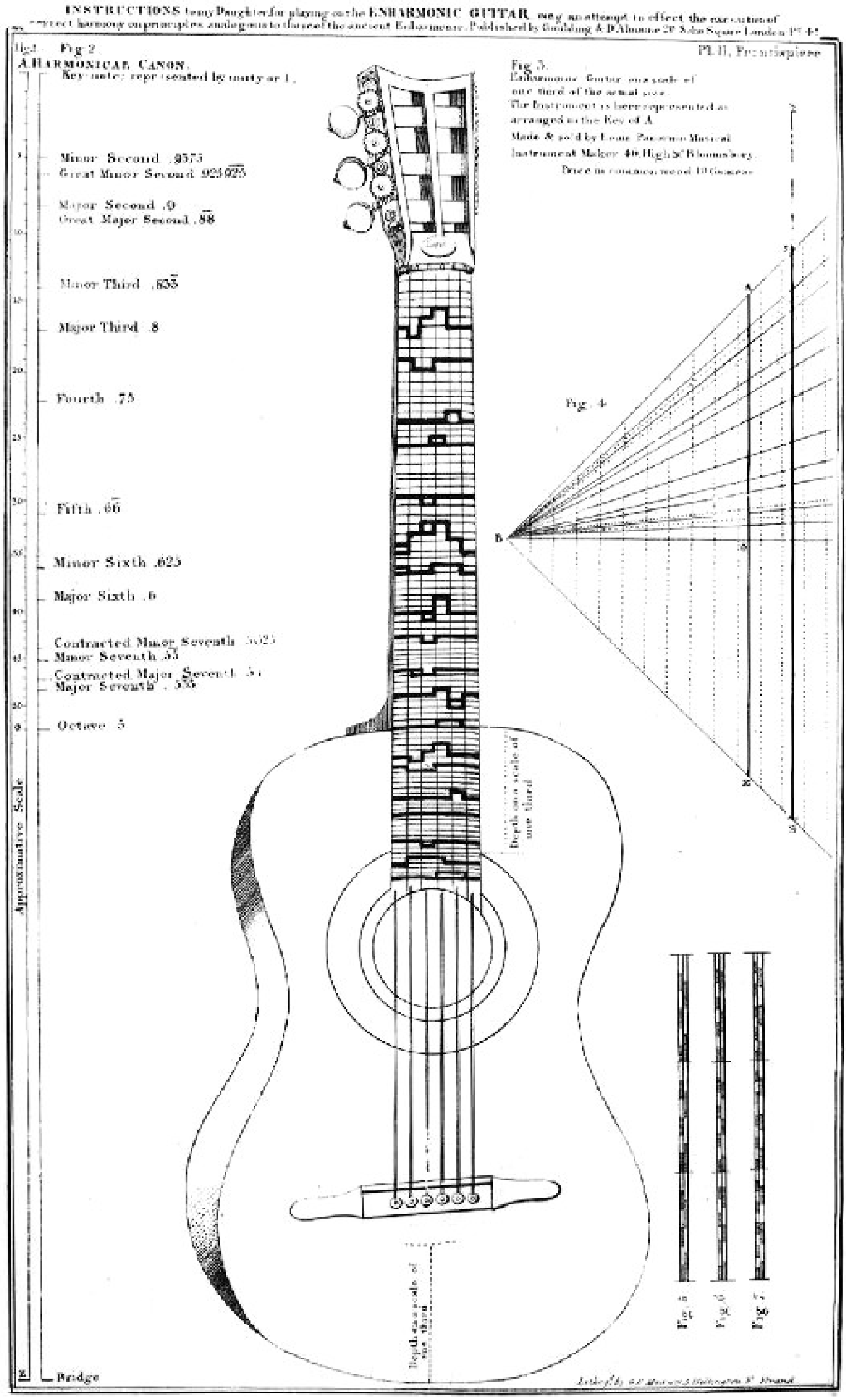


Fig 2
A HARMONICAL CANON.
Pl. II, Frontispiece
Fig 3.
arranged in the Key of A
Minor Second .9375
Great Minor Second .925925
Major Second .9
Great Major Second .88
Minor Third .833
Major Third .8
Fourth .75
Fig. 4
Fifth .66
Minor Sixth .625
Major Sixth .6
Contracted Minor Seventh .5625
Minor Seventh .55
Contracted Major Seventh
Major Seventh .533
Octave .5
Approximative Scale
Bridge
Fig. 5
Fig. 6
Fig. 7

Fig. 4.1

J. S. Bach's organ tuning was quite close to modern Equal Temperament, but it is known that he was opposed to strict mathematical equality of the semitones. Nevertheless he is known to be the first one to appreciate Equal Temperament by writing his "We11 - Tempered Clavier" — a collection of 48 preludes and fugues, two in each of the twelve major and twelve minor keys. By "Well - Tempered" he probably meant some kind of modified Mean - Tone Temperament, because, as I mentioned before, his system was not strictly equally tempered. Nevertheless, this system made it unnecessary to retune the harpsichord for each new piece.

This passage from Neidhardt (1732) expresses some representative early eighteenth century views on Equal Temperament:

> "Most people do not find in this tuning what they seek. It lacks, they say, variety in the beating of its major thirds and, consequently, a heightening of emotion. In a triad everything sounds bad enough; but if the major thirds alone, are played, the former sound much too high, the latter much too low... Yet if oboes, flutes & the like, and also violins, lutes, gambas & the rest, were all arranged in this same [tuning], then the inevitable church- and chamber-pitch would blend together throughout in the purest [way]... Thus Equal Temperament brings with it its comfort and discomfort, like blessed matrimony."

Some people have expressed the opinion that players of instruments such as the violin and the trombone, as well as singers, use perfectly consonant intervals, instead of equally tempered ones, because they can vary pitch gradually. Experimental data show that this is not true: musicians tend to stick to Equal Temperament. This is perhaps a result of its long use. In fact, only a few musicians can actually notice the slight differences.

Many tuning systems, other than the four discussed in chapters two and three, have been created, but it seems that none of them has been generally used. Some of them subdivide the octave into other than twelve equal units. In the next chapter I will discuss why twelve is the best number, but I will also show that seventeen gives some pleasant results too.

In the chart below the four discussed tuning systems are listed and the intervals between the different degrees of their scales are compared. The numbers indicate the width of the intervals in cents.

Chart 4.1 Diatonic Scale

		Pythagorean Tuning	Just Intonation	Mean-Tone Temperament	Equal Temperament
	C				
semitone		90	112	117	100
	B				
whole tone		204	204	193	200
	A				
whole tone		204	182	193	200
	G				
whole tone		204	204	193	200
	F				
semitone		90	112	117	100
	E				
whole tone		204	182	193	200
	D				
whole tone		204	204	193	200
	C				

Chart 4.2 Chromatic Scale

		Pythagorean Tuning	Just Intonation	Mean-Tone Temperament	Equal Temperament
	C				
		90	112	117	100
	B				
		114	92	76	100
s	Bb				
		90	112	117	100
e	A				
		90	90	117	100
m	G#				
		114	92	76	100
i	G				
		90	112	117	100
t	F#				
		114	92	76	100
o	F				
		90	112	117	100
n	E				
		114	92	76	100
e	Eb				
		90	90	117	100
s	D				
		90	112	117	100
	C#				
		114	92	76	100
	C				

Black notes are indicated by bold dotted lines.

CHAPTER FIVE

Seventeen – Divided Tuning System

Today, near the end of the twentieth century, the century of the bravest attempts to introduce something new in music, something unusual is happening: composers seem to be drawing back, returning to the old ways of composing. This is perhaps because there is no room for going further. After dodecaphonic music, which makes use of all the twelve degrees of the scale, there seems to be no possibility to create a new compositional technique. In this chapter I will present a new tuning system, which I developed with a division of the octave into seventeen smallest intervals. This number is not arbitrary; it is based on mathematical considerations, which I will present later in this chapter.

An equally tempered system has a lot of advantages, as discussed in chapter three. That is the reason why in the process of searching for a new suitable division I use equal temperament. I also keep the octave perfectly in tune, as it is in all major tuning systems known so far, because if I did not, there would be no pattern to repeat itself, and therefore there would be no pitch organization-at all.

In a tuning system it is important that the ratios of all the intervals are as close to simple integer ratios as possible. Therefore I will proceed in the following way: I will list all the ratios of small integers, whose denominator is less than ten, and whose magnitude is between one and two, because all the ratios within an octave (a ratio of 2) are within the interval [1,2]; later I will try to divide the octave into different numbers of smallest intervals, and then compare the so obtained ratios to the small-integer ones first listed. The number of divisions that produces ratios that compare best with the simple ones will produce the best tuning system.

All the possible simple ratios with denominator less than ten are:

1/1 = 2/2 = 3/3 = ... = 9/9 = $\boxed{1.000}$

2/1 = 4/2 = 6/3 = ... = 18/9 = $\boxed{2.000}$

3/2 = 6/4 = 9/6 = 12/8 = $\boxed{1.500}$

4/3 = 8/6 = 12/9 = $\boxed{1.333}$

5/3 = 10/6 = 15/9 = $\boxed{1.667}$

5/4 = 10/8 = $\boxed{1.250}$		7/4 = 14/8 = $\boxed{1.750}$	
6/5 = $\boxed{1.200}$	7/5 = $\boxed{1.400}$	8/5 = $\boxed{1.600}$	9/5 = $\boxed{1.800}$
7/6 = $\boxed{1.167}$	11/6 = $\boxed{1.833}$	8/7 = $\boxed{1.143}$	9/7 = $\boxed{1.285}$
10/7 = $\boxed{1.429}$	11/7 = $\boxed{1.571}$	12/7 = $\boxed{1.714}$	13/7 = $\boxed{1.857}$
9/8 = $\boxed{1.125}$	11/8 = $\boxed{1.375}$	13/8 = $\boxed{1.625}$	15/8 = $\boxed{1.875}$
10/9 = $\boxed{1.111}$	11/9 = $\boxed{1.222}$	13/9 = $\boxed{1.444}$	14/9 = $\boxed{1.556}$
16/9 = $\boxed{1.778}$	17/9 = $\boxed{1.889}$		

All these ratios[1] are listed in the left column of chart 5.1. The ones surrounded by a red rectangle are the most consonant, (with least value of the integers), and therefore most important. For a tuning system to sound good they have to be included. (Today, in Equal Temperament,

(1) except for 1.000 and 2.000, which are present in all divisions, because they correspond to the unison and the octave, as mentioned before. The octave has to be included.

they correspond to P1, P8, P5 and P4). The next are the "green ratios"; it would be good if they are included. After them in order of importance "the blue" and "the black" ratios follow.

Chart 5.1

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
							1.090 (—)	1.080 (—)	1.072 (—)	1.065 (—)	1.059 (—)					1.042 (—)
1.111						1.104 (−11.0)										1.085 (—)
1.125					1.122 (−3.91)						1.122 (−3.91)					1.130 (7.83)
1.143				1.149 (8.83)					1.149 (8.83)	1.134 (−13.6)						
1.167								1.167 (0.00)								1.177 (15.5)
1.200			1.189 (−13.6)				1.189 (−13.6)			1.208 (11.5)	1.189 (−13.6)					
1.222						1.219 (−4.55)			1.231 (12.7)							1.226 (5.53)
1.250		1.260 (13.7)			1.260 (13.7)			1.260 (13.7)			1.260 (13.7)					
1.285							1.296 (14.8)			1.287 (2.69)						1.277 (−10.6)
1.333				1.320 (−18.0)		1.346 (16.2)			1.320 (−18.0)		1.335 (2.31)		1.346 (16.2)	1.320 (−18.0)		**1.330** **(−3.93)**
1.375								1.361 (−17.7)		1.370 (−6.3)						1.386 (13.4)
1.400	1.414 -17.5		1.414 -17.5		1.414 -17.5		1.414 -17.5		1.414 -17.5		1.414 -17.5					
1.429																
1.444								1.470 (30.1)		1.459 (17.8)						1.443 (−1.32)
1.500				1.516 (18.0)		1.486 (−16.2)			1.516 (18.0)		**1.498** **(−2.31)**		1.486 (−16.2)	1.516 (18.0)		**1.503** **(3.93)**
1.556							1.554 (−2.23)			1.554 (−2.23)						1.566 (11.6)
1.571																
1.600		1.587 (−13.7)			1.587 (−13.7)			1.587 (−13.7)			1.587 (−13.7)					
1.625						1.641 (4.55)			1.625 (0.00)							1.631 (6.53)
1.667			1.682 (15.6)				1.682 (15.6)			1.656 (−11.5)	1.682 (15.6)					
1.714								1.714 (0.00)								1.699 (−15.1)
1.750				1.741 (−8.83)					1.741 (−8.83)							
1.778					1.782 (3.91)					1.763 (−14.7)	1.782 (3.91)					1.770 (−8.07)
1.800						1.811 (11.0)										
1.833							1.834 (0.94)									1.843 (9.78)
1.857								1.852 (−4.67)								
1.875									1.866 (−8.32)	1.878 (2.77)						
1.899											1.888 (−10.0)					
																1.920 (—)

Let us now take a more extensive look at chart 5.1. On the upper line the numbers 1, 2, 3, etc. are listed. They stand for the number of smallest intervals into which the octave is divided. Under each of these numbers the ratios obtained by this division are listed and how they compare to the small-integer ratios (SIR) to the left is indicated. Their colour is the colour of the SIR, to which they compare. Under each of these there is the result of the comparison, the difference in cents, written in. parentheses. It is negative, if the ratio of the degree is flat compared to the corresponding SIR, and positive if it is sharp. The number is written in red if it is smaller than 5 c., which means that the ratios compared are close to each other. I consider a tuning system a good one, if it possesses the following properties:

a) All the degrees have to be less than 18.0 c. off (17.5 is how much the tritone today is off.) There are exceptions only for the first one or two degrees of the scale, because they are always dissonant. For example, m2 today is -83 c. off. On the chart this dissonance of the first degrees is indicated with a "-".

b) The tuning system has to include the two most consonant ratios: the red ones 1.500 and 1.333.

Now, when one looks at the chart, one sees that only 5, 7, 10, 12, 14, 15, and 17 have the properties listed above. These numbers are written in red on the chart.

However, for 5, 10, and 15, the two red ratios are 18.0 cents off, which is too much for them to be consonant. Today, in the twelve-divided equally tempered system, the red ratios are only 2.31 c. off. 7 and 14 do not work for the same reasons: 16.2 c. off is too much for them to be consonant.

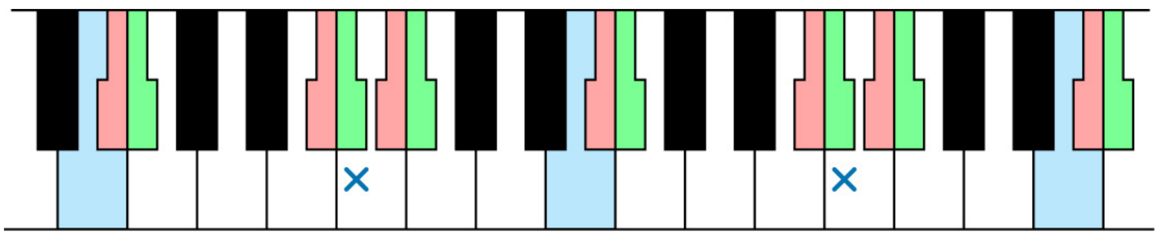
There are only two possibilities left: 12 and 17. (Twenty-four is also a possibility, but in this case the difference between two successive degrees would not be distinguishable.) Twelve is in fact the system used today, whose virtues have already been discussed. Let us now consider the seventeen-divided tuning system, the one I chose to develop.

It includes the two red ratios, and they are just as well approximated as in the twelve-divided system. However, there is the tritone in the twelve-divided system, and it is 17.5 c. off. In the seventeen-divided system in its place there is a degree, which is only 1.32 c. off, (boxed in the chart), which is much more consonant than the tritone, as shown on the enclosed tape. In fact, most of the degrees are less than 10 c. off, which will certainly have the overall effect of a better sound. The greatest advantage of the seventeen-divided tuning system is perhaps the fact that it provides more room for compositional effects: e.g. a much richer sound, due to the larger number of combinations of tones possible. In addition, this system can provide room for further development of music. As an example of the larger variety, there are three different triads in the place of the major and the minor ones today: the major, the minor-major, and the minor ones. (On the tape they are numbered 3, 5, and 7.) The augmented and the diminished triads are even more numerous.

Now, that the seventeen-divided tuning system has been created, the question how to apply it to practice has to be answered. The "white-key scale" has to be designed. There are two ways to do this:

5.1 Seven - Degree Scale Version

In this version there are seven white keys in an octave, just like in the twelve-divided tuning system used today. The piano keyboard in this case looks like this:



The blue keys are the basic tones of the octave, the blue cross indicates the 1.500 ratio (P5 today). The white-key scale can be heard on the tape. This version has two major disadvantages:

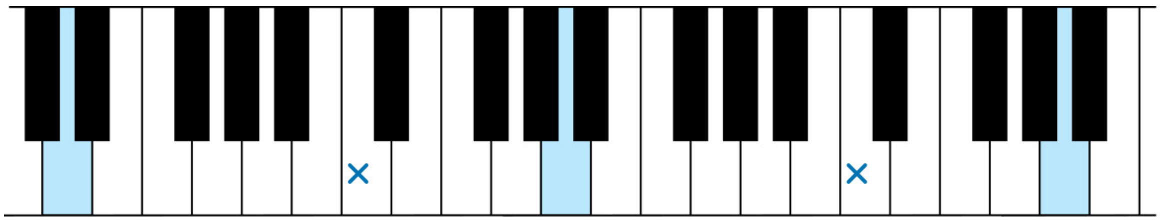
a) Between some white keys there are two "black" keys, (red and green on the figure), which will create great problems for the pianist. Similar problems exist for other instruments too.

b) Since it is very similar to the system today, listeners will find it out of tune, because there is a noticeable difference between these systems. In fact, on the tape the white-key scale does sound out of tune.

Therefore, I think that the ten - degree scale version is preferable.

5.2 Ten - Degree Scale Version

It has ten white keys in an octave and the piano keyboard looks approximately like this:



In the seventeen-divided tuning system the following degrees (intervals) are present:

Chart 5.2

Name:	perfect unison	dimin. second	second	third	augm. third
Ratio:	$2^{\frac{0}{17}} = 1$	$2^{\frac{1}{17}}$	$2^{\frac{2}{17}}$	$2^{\frac{3}{17}}$	$2^{\frac{4}{17}}$

Name:	fourth	augm. fourth	perfect fifth	dimin. sixth	sixth
Ratio:	$2^{\frac{5}{17}}$	$2^{\frac{6}{17}}$	$2^{\frac{7}{17}}$	$2^{\frac{8}{17}}$	$2^{\frac{9}{17}}$

Name:	perfect seventh	dimin. eighth	eighth	ninth	augm. ninth
Ratio:	$2^{\frac{10}{17}}$	$2^{\frac{11}{17}}$	$2^{\frac{12}{17}}$	$2^{\frac{13}{17}}$	$2^{\frac{14}{17}}$

Name:	tenth	augm. tenth	perfect eleventh
Ratio:	$2^{\frac{15}{17}}$	$2^{\frac{16}{17}}$	$2^{\frac{17}{17}} = 2$

The perfect fifth corresponds to P4 in the tuning system today, and the perfect seventh and perfect eleventh correspond to P5 and P8 respectively.

Now I will give explanations to the attached tape. On the first section one will hear:

a) the chromatic scale of the seventeen-divided tuning system;

b) the seven-degree and the ten-degree scale versions;

c) all possible intervals within an octave, both harmonically and melodically;

d) nine possible triads as follows:

Chart 5.3 "Diminished triads"

#	1	2	3
interval between second and first tine of triad	augm. third	augm. third	augm. third
interval between third and first tone of triad	dimin. sixth	sixth	perfect seventh

"Normal triads"			"Augmented triads"		
4	5	6	7	8	9
fourth	fourth	fourth	augm. fourth	augm. fourth	augm. fourth
sixth	perfect seventh	dimin. eighth	perfect seventh	dimin. eighth	eighth

All of these were recorded with the help of three audio generators, with frequency of the basic tone 392 Hz ("G").

On the second section of the tape one can hear:

1) The chromatic scale

2) The seven-degree scale version

3) The ten-degree scale version

4} All possible intervals

5) Triads # 3 , 5 and 7

6} All of the triads listed in chart 5.3

The second section was recorded with the help of guitar strings in order to give a more "real" (not "electrical ") sound. They are arranged in an instrument, constructed by myself (Fig. 5.3).

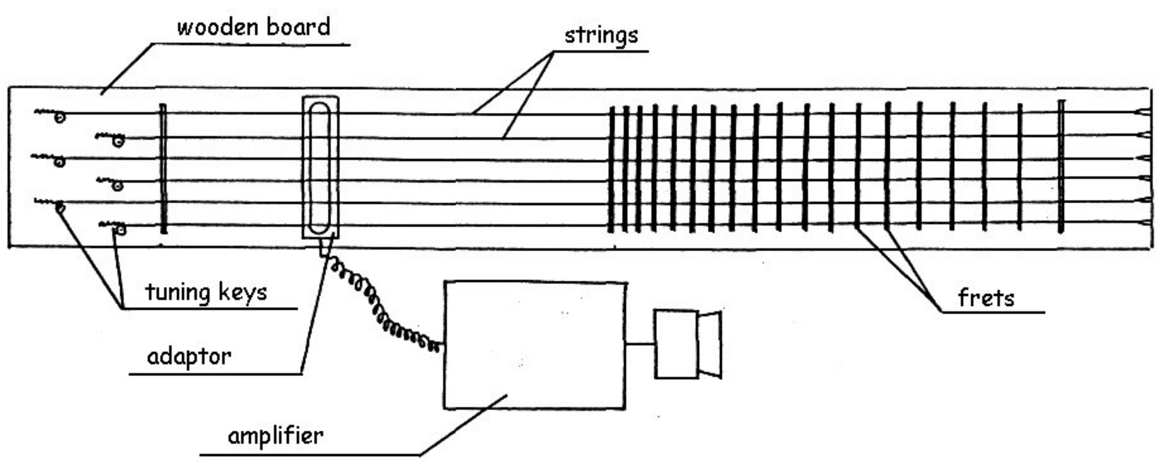


Fig. 5.3

CHAPTER SIX

Conclusions

There are several very important things to conclude. The first one is that equal temperament is absolutely necessary today, and to achieve it the perfectly in tune intervals have to be sacrificed. From ail possible divisions of the octave, only twelve and seventeen are suitable. The twelve- one is the one used today, while the seventeen- one is the one I developed in this essay and suggest as a possible branch of music.

The introduction of the seventeen-divided system, however, will be very difficult for several reasons:

a) People's ears are not used to it and therefore this system will have to overcome enormous opposition before it gains some recognition. Therefore it may take a long time to introduce it in practice.

b) This system will require totally new instrumentation.

c) None of the music ever written will be possible to be played in this system. This fact once again shows that it will take a long time to get it in use, as it has to stand the test of time. This also applies to the compositions being written in the seventeen-divided system. However, if one thinks of it as a new branch of music, while preserving the old music as well, he can be optimistic: many problems exist, but it is something new, and it creates more opportunity for composition with its much richer and varied sound. Hence, if there are enough optimistic contemporary composers, interested in experimenting with the new tuning system, it may succeed.

Tuning systems are an aspect of music, about which we often forget. I hope, that I have managed to show in this essay their great significance in the history of music, as well as in the history of man, since music is closely related to his mentality and culture.

* * *

Bibliography:

- Kvant magazine (Russian), April 1986, "Harmony In Music"
- Encyclopaedia Britanica, Macropaedia, Knowledge in depth; "Tuning And Temperament, Musical", v.18, pp.741
- The New GROVE Dictionary of Music And Musicians, Macmillan, 1980:
  "Just (Pure) Intonation", v.9, pp.756
  "Mean - Tone", v.11, pp.375
  "Pythagorean Intonation", v.15, pp.485
  "Temperaments", v.18, pp.660
- The Oxford Companion of Music, Oxford University Press, Tenth edition, 1970; "Temperament", pp.1012